NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4396

A PHENOMENOLOGICAL THEORY FOR THE TRANSIENT CREEP

OF METALS AT ELEVATED TEMPERATURES

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SUMMARY

The phenomenological theory previously proposed in NACA Technical Note 4000 for the behavior of metals at elevated temperatures has been modified to yield transient creep curves by assuming that the metal consists of two phases, each with its own elasticity and viscosity. The extended theory satisfies the basic requirements for a theory of transient creep at elevated temperatures: that the transient creep be closely connected with the subsequent steady creep, and that the apparent exponent of the time in the transient region be permitted wide variations between 0 and 1. From this theory it is possible to construct nondimensional creep curves which extend continuously from the transient region into the steady-state region. The corresponding family of creep curves for any metal may be obtained from the nondimensional family by use of appropriate constants. The constants required are those obtained from steady creep measurements, together with two additional constants which represent the difference between the phases. The transient creep curves resulting from this theory are compared with the experimental curves for pure aluminum, gamma iron, lead, and 7075-T6 aluminum alloy; good agreement is found.

INTRODUCTION

In studies of the creep of metals, it has become apparent that the creep behavior at elevated temperatures is different from that at low temperatures. For relatively pure metals the line of demarcation occurs at a temperature about one-half that at the melting point. In the high temperature region it has been shown that many aspects of the elevated-temperature behavior of some polycrystalline metals may be predicted if certain data are available concerning their steady creep. (See ref. 1.)

The transient or primary creep which takes place before the onset of the steady creep is not, however, one of the aspects of metal behavior which is predictable from the theory of reference 1. This transient component of the creep could be very important in some applications. An

understanding of the nature of transient creep would be highly desirable not only from the academic point of view but also because of the possibility of predicting the magnitude of the transient creep.

The existing theories of transient creep are inadequate to account for the actual behavior of polycrystalline metals at elevated temperatures. The so-called "exhaustion theory" of Mott and Nabarro (ref. 2) is a mechanistic theory involving the motion of dislocations and applies only to low stress and temperatures. The theory suggested by Orowan (ref. 3) is a semimechanistic theory which involves the use of thermal fluctuations to facilitate creep but also requires that the creep strain be proportional to the cube root of the time. A more recent theory of Mott (ref. 4), which also involves the movement of dislocations but is intended to apply to higher temperatures, likewise requires that the transient creep strain be proportional to the cube root of the time. Although some experimental data do indicate an apparent cube root relation, this result usually is true for only a part of the time range. Actually, when the logarithm of the creep strain is plotted against the logarithm of the time, the apparent exponent of the time may vary anywhere from a value in the neighborhood of one-sixth to unity.

One feature which seems to be characteristic of all the existing theories is that the mechanism which is supposed to give rise to the transient creep is distinctly different from that which gives rise to the subsequent steady creep. On the other hand, the experimental work of Dorn and his associates (ref. 5) indicates a strong interconnection between the transient and the steady creep at elevated temperatures. When the creep strain is plotted against an appropriate parameter, a single curve is obtained regardless of whether the creep is in the transient or the steady region. If the mechanism for transient creep were different from that for steady creep, one would expect two different temperature dependencies and consequently two curves instead of one would be required. Thus, any proposed theory of transient creep which is intended to apply at elevated temperatures must satisfy at least two elementary requirements: It must show a close connection with the steady creep which follows, and it must permit the apparent exponent of the time in the transient region to vary over a wide range between 0 and 1.

In the treatment of the elevated-temperature behavior of metals in reference 1, the physical properties necessary to describe the behavior at constant temperature were elasticity and viscosity. The metal was thus conceived to consist of a single phase specified by the elasticity and viscosity constants. Suppose, however, that the metal could be more accurately described as consisting of two phases, each with its own set of elasticity and viscosity constants. Then, with such a metal, it is conceivable that, after application of a stress, the two phases would come to action at different rates and would thus result in an overall creep rate which would vary with the time; or, the metal would exhibit transient creep.

This paper is an attempt to account for the experimental phenomena of transient creep at elevated temperatures by considering the metal to consist of two phases, each of which satisfies the phenomenological relation of reference 1 separately. On this basis transient creep arises simply because the two phases have different constants. The advantages of the phenomenological approach are retained by this theory and no new mechanism is needed or postulated to account for the transient creep.

After the presentation of the theory for transient creep, a comparison is made between the results from the theory and experimental data obtained from four metals at absolute temperatures above about one-half the melting point.

SYMBOLS

ϵ creep strain

$\dot{\epsilon}$ creep strain rate, per hour

σ stress, psi

$\bar{\sigma}$ applied stress, psi

σ_0 constant, psi

E elastic tensile modulus, psi

T temperature, $^{\circ}K$

M a temperature function of the form $2sTe^{-\Delta H/RT}$, per hour

s constant, per hour per $^{\circ}K$

ΔH activation energy, calorie/mol

R gas constant, taken as 2 calorie/mol/$^{\circ}K$

τ time constant, hr

$$r = \tanh\left[\frac{1}{2}\left(\frac{2E_1}{E_1 + E_2}\frac{\bar{\sigma}}{\sigma_0} - \tanh^{-1}\frac{M_2 \sinh\frac{2\bar{\sigma}}{\sigma_0}}{M_1 + M_2 \cosh\frac{2\bar{\sigma}}{\sigma_0}}\right)\right]$$

t   time, hr

$\theta = te^{-\Delta H/RT}$, hr

Subscripts 1 and 2 apply to phases 1 and 2, thus, $M_1 = 2s_1 Te^{-\Delta H/RT}$

THEORY FOR TRANSIENT CREEP

Previous work on the behavior at elevated temperatures of metals (ref. 1) has shown that much of their behavior is predictable if the metal is considered to be an elastic-viscous substance satisfying (at constant temperature) a relation of the type

$$\text{Strain rate} = \frac{1}{E}\frac{d}{dt}(\text{Stress}) + M \sinh\left(\frac{\text{Stress}}{\text{Constant}}\right)$$

where M is a known function of the temperature and E is the elastic modulus. The constants are determined from measurements of the steady creep rates at constant stress and temperature. The use of this relation implies that the elastic-viscous material is uniform and all of one phase. Under conditions of constant stress, this simple relation can yield only a constant creep rate.

Suppose, however, that two phases were present in the metal, each with its own set of elasticity and viscosity constants. One phase, for example, might possess considerably more elastic stiffness than the other (different values of E), or they might have widely different viscous properties (different values of M), or both. In such a case, under conditions of constant stress, the interaction between the phases would result in a variable creep rate.

Assume a polycrystalline metal to consist of two such phases (designated by subscripts 1 and 2). The phenomenological relation proposed in reference 1 for metals at elevated temperatures is here assumed to hold for each phase separately (with temperature constant):

$$\left.\begin{aligned}
\dot{\epsilon}_1 &= \frac{1}{E_1}\frac{d\sigma_1}{dt} + M_1 \sinh\frac{\sigma_1}{\sigma_0} \\[2ex]
\dot{\epsilon}_2 &= \frac{1}{E_2}\frac{d\sigma_2}{dt} + M_2 \sinh\frac{\sigma_2}{\sigma_0}
\end{aligned}\right\} \tag{1}$$

where σ_1 and σ_2 are the stresses borne by the individual phases.

Indirect experiméntal evidence, to be discussed later, indicates strongly that the two assumed phases are present in the metal in equal or nearly equal amounts. If this is so, the following relation must hold for equilibrium:

$$\frac{\sigma_1 + \sigma_2}{2} = \bar{\sigma} \tag{2}$$

where $\bar{\sigma}$ is the applied stress. Also, since the metal must retain its continuity, it follows that

$$\left.\begin{aligned} \epsilon_1 &= \epsilon_2 \\ \dot{\epsilon}_1 &= \dot{\epsilon}_2 \end{aligned}\right\} \tag{3}$$

The relation between the moduli satisfying equations (2) and (3) is

$$E = \frac{E_1 + E_2}{2}$$

Details of the solution to this problem are presented in the appendix. There it is shown that the creep strain developed at a time t after the application of a constant stress $\bar{\sigma}$ is

$$\epsilon = \sigma_0 \frac{\left(\dfrac{\frac{M_1}{M_2} + \cosh\frac{2\bar{\sigma}}{\sigma_0}}{E_2} - \dfrac{\frac{M_2}{M_1} + \cosh\frac{2\bar{\sigma}}{\sigma_0}}{E_1}\right)\left[2\tanh^{-1}r - 2\tanh^{-1}\!\left(re^{-t/\tau}\right)\right] + \left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)\sinh\dfrac{2\bar{\sigma}}{\sigma_0}\log\dfrac{\sinh\left(2\tanh^{-1}r\right)}{\sinh\left[2\tanh^{-1}\left(re^{-t/\tau}\right)\right]}}{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}} \tag{4}$$

where

$$r = \tanh\left[\frac{1}{2}\left(\frac{2E_1}{E_1 + E_2}\frac{\bar{\sigma}}{\sigma_0} - \tanh^{-1}\frac{M_2\sinh\dfrac{2\bar{\sigma}}{\sigma_0}}{M_1 + M_2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}}\right)\right]$$

$$\tau = \frac{\dfrac{\sigma_0}{\sqrt{M_1 M_2}}\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)}{\sqrt{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}}}$$

It is evident by inspection that the left-hand term in the numerator of the expression for ϵ will vanish if the phases have the same constants;

that is, if $\dfrac{M_1}{M_2} = \dfrac{E_1}{E_2} = 1$. It is shown in the appendix that under this condition the only creep possible is a steady creep proportional to the time.

The magnitude of the creep as a function of the time therefore depends on the values of the two parameters $\dfrac{M_1}{M_2}$ and $\dfrac{E_1}{E_2}$. These values are unknown, but two limiting cases may be considered:

(a) The transient creep will be a maximum when the two parameters $\dfrac{M_1}{M_2}$ and $\dfrac{E_1}{E_2}$ are as different from unity as possible. Suppose as the limiting case that $\dfrac{E_1}{E_2} = \infty$. Then equation (4) becomes

$$\epsilon = \frac{\sigma_0}{E_2}\,\frac{\left(\dfrac{M_1}{M_2} + \cosh\dfrac{2\bar{\sigma}}{\sigma_0}\right)\left[2\tanh^{-1}r - 2\tanh^{-1}\!\left(re^{-t/\tau}\right)\right] + \sinh\dfrac{2\bar{\sigma}}{\sigma_0}\log\dfrac{\sinh\!\left(2\tanh^{-1}r\right)}{\sinh\!\left[2\tanh^{-1}\!\left(re^{-t/\tau}\right)\right]}}{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}} \tag{5a}$$

(b) The transient creep will be a minimum when the two parameters $\dfrac{M_1}{M_2}$ and $\dfrac{E_1}{E_2}$ are as close to unity as possible. Suppose as the limiting case that $\dfrac{E_1}{E_2} = 1$. Then equation (4) becomes

$$\epsilon = \frac{\sigma_0}{E}\,\frac{\left(\dfrac{M_1}{M_2} - \dfrac{M_2}{M_1}\right)\left[2\tanh^{-1}r - 2\tanh^{-1}\!\left(re^{-t/\tau}\right)\right] + 2\sinh\dfrac{2\bar{\sigma}}{\sigma_0}\log\dfrac{\sinh\!\left(2\tanh^{-1}r\right)}{\sinh\!\left[2\tanh^{-1}\!\left(re^{-t/\tau}\right)\right]}}{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}} \tag{5b}$$

where $E = E_1 = E_2$.

In both cases the creep strain may be plotted as a function of the time for different values of the parameter $\dfrac{M_1}{M_2}$. One method of presenting the data is to plot the creep strain as a function of $\theta = te^{-\Delta H/RT}$, a parameter which has had extensive experimental verification for pure

metals. It is shown in the appendix that in the notation of this paper

$$\theta = \frac{\frac{t}{\tau} \sigma_0 \left(\frac{1}{E_1} + \frac{1}{E_2} \right)}{2sT \sqrt{\frac{M_1}{M_2} + \frac{M_2}{M_1} + 2 \cosh \frac{2\bar{\sigma}}{\sigma_0}}}$$ (6)

From equations (5a), (5b), and (6) nondimensional plots have been constructed which show the maximum $\left(\text{fig. 1(a) for } \frac{E_1}{E_2} = \infty \right)$ and the minimum $\left(\text{fig. 1(b) for } \frac{E_1}{E_2} = 1 \right)$ possible theoretical transient creep strain for values of $\frac{2\bar{\sigma}}{\sigma_0}$ equal to 1, 5, 10, 15, 20, and 25.

These families of curves are universal in the sense that the theoretical transient creep characteristics of any material can be obtained from them by using the constants appropriate to that material to transform the coordinates to ϵ, θ in each case. This procedure was followed in the comparison of theory and experiment discussed in the next section.

COMPARISON OF THEORY AND EXPERIMENT

Experimental data sufficiently comprehensive for use in comparison with theoretical results are available for the creep of three nearly pure metals: aluminum, gamma iron, lead, and for 7075-T6 aluminum alloy. In each case the data covered the transient as well as the steady creep region. Data in the latter region are required in order to determine the necessary constants.

The numerical values of the experimental constants from steady creep data (taken from refs. 1, 5, 6, 7, 8, and 9) and the constants assumed for the calculations are given in table I. (The value of 2sT in this table is considered to vary so slowly with T that it is taken as constant at a value of T in the middle of the temperature range.) For the aluminum alloy, reference 1 gives only information on steady creep, inasmuch as the transient creep data are unpublished.

Because the theoretical curves are calculated for ratios $\frac{2\bar{\sigma}}{\sigma_0}$ equal to 1, 5, 10, 15, 20, or 25, the corresponding experimental curves were

selected wherever possible with values of $\frac{2\bar{\sigma}}{\sigma_0}$ closest to these numbers.

The selection of the ratios $\frac{M_1}{M_2}$ and $\frac{E_1}{E_2}$ was made by trial and error, since there is no direct way of determining these ratios. Also, since the value of σ_0 is known from steady creep measurements, and since a value of $\frac{\sigma_0}{E_2}$ $\left(\text{or } \frac{\sigma_0}{E}\right)$ must be assumed for obtaining the theoretical curves, a value of E_2 (or E) is implied. Table II lists the values of E_2 implied by the calculations $\left(E_2 \text{ is the same as } E \text{ for the aluminum alloy}\right)$ and compares these values with the corresponding values of E_1; the resulting ratios of E_1 to E_2 must be such as to justify the use of figure 1(a) or of figure 1(b), as the case may be. For the pure metals the ratio $\frac{E_1}{E_2}$ was considerably larger than unity and thus the use of figure 1(a) in obtaining the theoretical curves was justified; for the aluminum alloy, the ratio was of the order of unity and the use of figure 1(b) was justified.

Figures 2, 3, and 4 show the comparisons of the experimental curves for the three nearly pure metals with the corresponding theoretical curves determined from figure 1(a), made by using the constants listed in the right-hand columns of table I. In all cases the constants used were the same as those measured for the steady-state condition. The agreement with the experimental curves is good in all cases and is especially satisfactory in the case of pure aluminum.

Figure 5 shows a similar comparison for the 7075-T6 aluminum alloy. In this case the theoretical curves were taken from figure 1(b). This comparison is of special interest because the elevated-temperature behavior of 7075-T6 aluminum alloy has been treated in detail in a previous paper. (See ref. 1.) The constants selected give a family of theoretical curves which match the experimental curves well at the higher stresses; the match is not so good for the very lowest stress $\left(\frac{2\bar{\sigma}}{\sigma_0} = 5\right)$.

The sudden upward change in curvature at the right-hand end of some of the experimental curves, representing the onset of tertiary creep, cannot, of course, be given by a theory which has no concern with tertiary creep.

DISCUSSION

Examination of equation 5(a) or 5(b) shows that the theoretical stress parameter for the transient condition is $\dfrac{\bar{\sigma}}{\dfrac{\sigma_0}{2}}$. The corresponding parameter for the steady state is known to be $\dfrac{\bar{\sigma}}{\sigma_0}$. (See ref. 1.) Thus there is an apparent increase in the denominator of the stress parameter by a factor of 2 in passing from the transient to the steady state. This factor of 2 is a direct consequence of the original assumption implicit in equation (2) that the two phases are present in equal amounts.

An interesting experimental confirmation of this change in the denominator is found in reference 5. In table II of that reference the numerical value of the denominator (listed as $1/B$) is given as 191 pounds per square inch for pure aluminum under transient conditions. A study of figure 3 of that reference discloses, however, that the value of $Z = \dot{\varepsilon}e^{\Delta H/RT}$ for the steady state is given by $\left(0.5 \times 10^{11}\right)\left(\sinh \dfrac{\bar{\sigma}}{382}\right)\text{hr}^{-1}$. This relation indicates a value of 382 pounds per square inch for the denominator after the creep rate has become constant. The doubling of the denominator is thus a fact and gives powerful support to the assumption of phases present in equal amounts.

Transient creep in this theory results from the two phases coming into action at different rates after an external stress is applied to the metal. The mechanism is the same as that which ultimately gives rise to the steady creep embodied in the phenomenological relation of reference 1. The theory thus satisfies the requirement mentioned in the "Introduction" that there be an intimate connection between the transient and the steady creep in the elevated-temperature region. That it also satisfies the other requirement - a wide range of values for the apparent exponent of the time - may be seen by a casual inspection of figures 1(a) and 1(b). It is worthy of note that these apparent exponents do actually give the illusion of being constant over an appreciable range of time in many cases.

McLean, in his study of the creep of pure aluminum, was able to isolate the different effects which made up the total creep and presented curves which showed the individual contributions of the grain boundary shearing, slip, and grain rotation. Figure 6(a), taken from reference 10, is a typical set of curves of this kind and shows that the grain boundary shearing comes to a stop after a time, while the other components behave in a viscous manner and continue to contribute to the creep. In the present paper, the creep strain given by equation (5a) is made up of

two terms; figure 6(b) shows the individual contributions from each term. Note the similarity to figure 6(a). The formal correspondence between the two figures does not necessarily imply that the lower theoretical curve is concerned exclusively with grain boundaries, but it is regarded as significant that the theory for a two-phase material yields curves similar to those observed in a polycrystalline metal.

A question of interest suggested by this theory concerns the distribution of stress between the phases. Figure 7 shows the distribution of stress between the phases, calculated from equation (A15) of the appendix for two values of the stress ratio $\frac{2\bar{\sigma}}{\sigma_0}$. These curves show that initially most of the stress is borne by the metal of phase 1 and that, as time passes, more and more of the stress is taken up by the metal of phase 2, until after a considerable time $(t \gg \tau)$ the loads carried by the two phases are roughly comparable and constant. (The loads are exactly equal if $M_1 = M_2$.) The higher the ratio $\frac{2\bar{\sigma}}{\sigma_0}$, the more evenly the load is divided between the phases when the creep becomes steady.

The difference in the values of $\frac{E_1}{E_2}$ and $\frac{M_1}{M_2}$ for the pure metals and for the aluminum alloy is noteworthy. The pure metals require that $\frac{E_1}{E_2}$ be much greater than unity whereas the alloy calls for a value of $\frac{E_1}{E_2} = 1$. At the same time, the value of $\frac{M_1}{M_2}$ for the alloy is larger than that for the pure metals.

The nondimensional curve families have been constructed for only two values of $\frac{E_1}{E_2}$, that is, ∞ and 1. It is believed that the curves for any value of $\frac{E_1}{E_2}$ greater than perhaps 10 would not differ too greatly from those for $\frac{E_1}{E_2} = \infty$. If it should prove that values of $\frac{E_1}{E_2}$ between 1 and 10 are encountered, it might be well to construct an intermediate set. The limited experience with the theory to date has not yielded such values.

CONCLUDING REMARKS

A method has been presented by which transient creep curves for any material at elevated temperatures may be derived from a set of universal, nondimensional transient creep curves. The constants which are required for the conversion are those obtained from measurements on steady creep alone and two other parameters $\dfrac{M_1}{M_2}$ and $\dfrac{E_1}{E_2}$ where M denotes a temperature function and E, the elastic modulus for each phase.

The comparison between the theoretical curves obtained in this way with experimental data on pure aluminum, gamma iron, lead, and 7075-T6 aluminum alloy shows that the theory is adequate to account for the shape and position of the experimental curves. For each metal, the assumption of a single set of constants yields the entire family of transient creep curves properly spaced in regard to stress and temperature and having substantially the required slope.

Langley Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., July 18, 1958.

APPENDIX

DEVELOPMENT OF FORMULAS FOR TRANSIENT CREEP

Differential Equations of System

Let the polycrystalline metal consist of two phases designated by subscripts 1 and 2. The differential equations for the system are taken to be, at constant temperature, two relations identical in form with the relation proposed in reference 1:

$$\dot{\epsilon}_1 = \frac{1}{E_1} \frac{d\sigma_1}{dt} + M_1 \sinh \frac{\sigma_1}{\sigma_0} \tag{A1}$$

$$\dot{\epsilon}_2 = \frac{1}{E_2} \frac{d\sigma_2}{dt} + M_2 \sinh \frac{\sigma_2}{\sigma_0} \tag{A2}$$

For compatibility between the phases, the relations

$$\epsilon_1 = \epsilon_2 \tag{A3}$$

$$\dot{\epsilon}_1 = \dot{\epsilon}_2 \tag{A4}$$

must hold and for equilibrium

$$\frac{\sigma_1 + \sigma_2}{2} = \bar{\sigma} \tag{A5}$$

where $\bar{\sigma}$ is the constant applied stress.

One of the stresses, for example, σ_2, may be eliminated by means of equation (A5); a differential equation for σ_1 results:

$$\frac{d\sigma_1}{dt} + \frac{M_1 + M_2 \cosh \frac{2\bar{\sigma}}{\sigma_0}}{\frac{1}{E_1} + \frac{1}{E_2}} \sinh \frac{\sigma_1}{\sigma_0} - \frac{M_2 \sinh \frac{2\bar{\sigma}}{\sigma_0}}{\frac{1}{E_1} + \frac{1}{E_2}} \cosh \frac{\sigma_1}{\sigma_0} = 0 \tag{A6}$$

Solution for Stresses

Equation (A6) has a solution

$$\tanh\left[\frac{1}{2}\left(\frac{\sigma_1}{\sigma_0} - \tanh^{-1}\frac{M_2 \sinh\frac{2\bar{\sigma}}{\sigma_0}}{M_1 + M_2 \cosh\frac{2\bar{\sigma}}{\sigma_0}}\right)\right] = \tanh\left[\frac{1}{2}\left(\frac{E_1}{E_1+E_2}\frac{2\bar{\sigma}}{\sigma_0} - \tanh^{-1}\frac{M_2 \sinh\frac{2\bar{\sigma}}{\sigma_0}}{M_1 + M_2 \cosh\frac{2\bar{\sigma}}{\sigma_0}}\right)\right]e^{-t/\tau} \qquad (A7)$$

where

$$\tau = \frac{\dfrac{\sigma_0}{\sqrt{M_1 M_2}}\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)}{\sqrt{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}}} \qquad (A8)$$

and in which

$$\left(\sigma_1\right)_{t=0} = \frac{2E_1}{E_1+E_2}\,\bar{\sigma} \qquad (A9)$$

is the initial condition.

Similarly, by interchanging subscripts, the corresponding solution for σ_2 is

$$\tanh\left[\frac{1}{2}\left(\frac{\sigma_2}{\sigma_0} - \tanh^{-1}\frac{M_1 \sinh\frac{2\bar{\sigma}}{\sigma_0}}{M_2 + M_1 \cosh\frac{2\bar{\sigma}}{\sigma_0}}\right)\right] = \tanh\left[\frac{1}{2}\left(\frac{E_2}{E_1+E_2}\frac{2\bar{\sigma}}{\sigma_0} - \tanh^{-1}\frac{M_1 \sinh\frac{2\bar{\sigma}}{\sigma_0}}{M_2 + M_1 \cosh\frac{2\bar{\sigma}}{\sigma_0}}\right)\right]e^{-t/\tau} \qquad (A10)$$

in which

$$\left(\sigma_2\right)_{t=0} = \frac{2E_2}{E_1+E_2}\,\bar{\sigma} \qquad (A11)$$

is the initial condition. Denote

$$a_1 = \tanh^{-1} \frac{M_2 \sinh \frac{2\bar{\sigma}}{\sigma_0}}{M_1 + M_2 \cosh \frac{2\bar{\sigma}}{\sigma_0}}$$

$$a_2 = \tanh^{-1} \frac{M_1 \sinh \frac{2\bar{\sigma}}{\sigma_0}}{M_2 + M_1 \cosh \frac{2\bar{\sigma}}{\sigma_0}}$$

(A12)

$$r_1 = \tanh\left[\frac{1}{2}\left(\frac{E_1}{E_1 + E_2}\frac{2\bar{\sigma}}{\sigma_0} - a_1\right)\right]$$

$$r_2 = \tanh\left[\frac{1}{2}\left(\frac{E_2}{E_1 + E_2}\frac{2\bar{\sigma}}{\sigma_0} - a_2\right)\right]$$

(A13)

In this notation the solutions for the stresses are:

$$\tanh\left[\frac{1}{2}\left(\frac{\sigma_1}{\sigma_0} - a_1\right)\right] = r_1 e^{-t/\tau}$$

$$\tanh\left[\frac{1}{2}\left(\frac{\sigma_2}{\sigma_0} - a_2\right)\right] = r_2 e^{-t/\tau}$$

(A14)

or

$$\frac{\sigma_1}{\sigma_0} = a_1 + 2\tanh^{-1}\left(r_1 e^{-t/\tau}\right)$$

$$\frac{\sigma_2}{\sigma_0} = a_2 + 2\tanh^{-1}\left(r_2 e^{-t/\tau}\right)$$

(A15)

It can be readily shown that

$$a_1 + a_2 = \frac{2\bar{\sigma}}{\sigma_0} \tag{A16}$$

From equations (A13) this relation makes

$$r_1 = -r_2 = r \tag{A17}$$

The second of equations (A15) may therefore be written

$$\frac{\sigma_2}{\sigma_0} = \frac{2\bar{\sigma}}{\sigma_0} - \left[a_1 + 2 \tanh^{-1}\!\left(re^{-t/\tau}\right) \right] = \frac{2\bar{\sigma}}{\sigma_0} - \frac{\sigma_1}{\sigma_0}$$

in accordance with condition (A5).

Solution for Strain Rate

Since the stresses are now known, the appropriate functions may now be inserted into equation (A1) or (A2) to obtain the strain rate. If the first of equations (A15) is used for the value of σ_1, it follows that

$$\frac{d\sigma_1}{dt} = - \frac{\sqrt{M_1{}^2 + M_2{}^2 + 2M_1M_2 \cosh \frac{2\bar{\sigma}}{\sigma_0}}}{\frac{1}{E_1} + \frac{1}{E_2}} \frac{2re^{-t/\tau}}{1 - r^2 e^{-2t/\tau}}$$

and that

$$\sinh \frac{\sigma_1}{\sigma_0} = \frac{\left(M_1 + M_2 \cosh \frac{2\bar{\sigma}}{\sigma_0}\right)\dfrac{2re^{-t/\tau}}{1 - r^2 e^{-2t/\tau}} + M_2 \sinh \frac{2\bar{\sigma}}{\sigma_0} \dfrac{1 + r^2 e^{-2t/\tau}}{1 - r^2 e^{-2t/\tau}}}{\sqrt{M_1{}^2 + M_2{}^2 + 2M_1M_2 \cosh \frac{2\bar{\sigma}}{\sigma_0}}}$$

Substitution of these quantities into equation (A1) gives for the strain rate

$$\dot{\epsilon}_1 = \frac{\left(\dfrac{M_1{}^2 + M_1M_2 \cosh \dfrac{2\bar{\sigma}}{\sigma_0}}{E_2} - \dfrac{M_2{}^2 + M_1M_2 \cosh \dfrac{2\bar{\sigma}}{\sigma_0}}{E_1}\right)2re^{-t/\tau} + \left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)M_1M_2 \sinh \dfrac{2\bar{\sigma}}{\sigma_0}\left(1 + r^2e^{-2t/\tau}\right)}{\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)\left(1 - r^2e^{-2t/\tau}\right)\sqrt{M_1{}^2 + M_2{}^2 + 2M_1M_2 \cosh \dfrac{2\bar{\sigma}}{\sigma_0}}} \tag{A18}$$

Note that, if the phases have identical properties $\left(E_1 = E_2 = E;\right.$ $\left. M_1 = M_2 = M\right)$ so that they become equivalent to a single phase, then

$$r = \tanh\left[\frac{1}{2}\left(\frac{\bar{\sigma}}{\sigma_0} - \tanh^{-1}\frac{\sinh \dfrac{2\bar{\sigma}}{\sigma_0}}{1 + \cosh \dfrac{2\bar{\sigma}}{\sigma_0}}\right)\right] = \tanh\left[\frac{1}{2}\left(\frac{\bar{\sigma}}{\sigma_0} - \frac{\bar{\sigma}}{\sigma_0}\right)\right] = 0$$

and under this condition

$$\dot{\epsilon}_1 = \frac{M \sinh \dfrac{2\bar{\sigma}}{\sigma_0}}{\sqrt{2 + 2\cosh \dfrac{2\bar{\sigma}}{\sigma_0}}} = \frac{M \sinh \dfrac{\bar{\sigma}}{\sigma_0} \cosh \dfrac{\bar{\sigma}}{\sigma_0}}{\cosh \dfrac{\bar{\sigma}}{\sigma_0}} = M \sinh \frac{\bar{\sigma}}{\sigma_0} \tag{A19}$$

which is a constant and of the correct form known for steady creep. Transient creep will therefore appear only as a result of difference between the phases.

Note also, that even if r has values different from zero, after an infinite time

$$\left(\dot{\epsilon}_1\right)_{t=\infty} = \frac{\sqrt{M_1M_2} \sinh \dfrac{2\bar{\sigma}}{\sigma_0}}{\sqrt{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh \dfrac{2\bar{\sigma}}{\sigma_0}}} \tag{A20}$$

which is likewise a constant; this relation shows that the end result is always steady creep. For stresses large enough so that the ratios $\dfrac{M_1}{M_2}$ and $\dfrac{M_2}{M_1}$ may be neglected in the denominator, which is the most usual case, then

$$(\dot{\epsilon})_{t=\infty} = \frac{\sqrt{M_1 M_2}\,\frac{1}{2}\,e^{2\bar{\sigma}/\sigma_0}}{e^{\bar{\sigma}/\sigma_0}} = \sqrt{M_1 M_2}\,\frac{e^{\bar{\sigma}/\sigma_0}}{2} \cong \sqrt{M_1 M_2}\,\sinh\frac{\bar{\sigma}}{\sigma_0}$$

The geometric mean $\sqrt{M_1 M_2}$ is thus identifiable as

$$\sqrt{M_1 M_2} = 2sTe^{-\Delta H/RT} \tag{A21}$$

from the known expression for steady creep. (See ref. 1.) Note that for the transient state the stress parameter is $\dfrac{\bar{\sigma}}{\dfrac{\sigma_0}{2}}$, whereas for the steady state the parameter is $\dfrac{\bar{\sigma}}{\sigma_0}$.

Solution for Creep Strain

Integration of equation (A18) yields the creep strain

$$\epsilon = \epsilon_1 = \tau \frac{\left\{\left(\dfrac{M_1^2 + M_1 M_2 \cosh\frac{2\bar{\sigma}}{\sigma_0}}{E_2} - \dfrac{M_2^2 + M_1 M_2 \cosh\frac{2\bar{\sigma}}{\sigma_0}}{E_1}\right)\left[2\tanh^{-1}r - 2\tanh^{-1}\left(re^{-t/\tau}\right)\right] + \left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)M_1 M_2 \sinh\frac{2\bar{\sigma}}{\sigma_0}\log\dfrac{\sinh\left(2\tanh^{-1}r\right)}{\sinh\left[2\tanh^{-1}\left(re^{-t/\tau}\right)\right]}\right\}}{\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)\sqrt{M_1^2 + M_2^2 + 2M_1 M_2 \cosh\frac{2\bar{\sigma}}{\sigma_0}}} \tag{A22}$$

in which $\left(\epsilon_1\right)_{t=0} = 0$ is the initial condition. The values of τ may be inserted from equation (A8) and the following equation results:

$$\epsilon = \sigma_0 \frac{\left\{\left(\dfrac{\dfrac{M_1}{M_2} + \cosh\frac{2\bar{\sigma}}{\sigma_0}}{E_2} - \dfrac{\dfrac{M_2}{M_1} + \cosh\frac{2\bar{\sigma}}{\sigma_0}}{E_1}\right)\left[2\tanh^{-1}r - 2\tanh^{-1}\left(re^{-t/\tau}\right)\right] + \left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)\sinh\frac{2\bar{\sigma}}{\sigma_0}\log\dfrac{\sinh\left(2\tanh^{-1}r\right)}{\sinh\left[2\tanh^{-1}\left(re^{-t/\tau}\right)\right]}\right\}}{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\frac{2\bar{\sigma}}{\sigma_0}} \tag{A23}$$

Two cases may be considered:

(a) Assume $\dfrac{E_1}{E_2} = \infty$. Under this condition

$$\epsilon = \frac{\sigma_0}{E_2} \frac{\left(\dfrac{M_1}{M_2} + \cosh \dfrac{2\bar{\sigma}}{\sigma_0}\right)\left[2 \tanh^{-1} r - 2 \tanh^{-1}\left(re^{-t/\tau}\right)\right] + \sinh \dfrac{2\bar{\sigma}}{\sigma_0} \log \dfrac{\sinh\left(2 \tanh^{-1} r\right)}{\sinh\left[2 \tanh^{-1}\left(re^{-t/\tau}\right)\right]}}{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2 \cosh \dfrac{2\bar{\sigma}}{\sigma_0}} \tag{A24a}$$

(b) Assume $\dfrac{E_1}{E_2} = 1$. Under this condition

$$\epsilon = \frac{2\sigma_0}{E} \frac{\left(\dfrac{M_1}{M_2} - \dfrac{M_2}{M_1}\right)\left[\tanh^{-1} r - \tanh^{-1}\left(re^{-t/\tau}\right)\right] + \sinh \dfrac{2\bar{\sigma}}{\sigma_0} \log \dfrac{\sinh\left(2 \tanh^{-1} r\right)}{\sinh\left[2 \tanh^{-1}(re^{-t/\tau})\right]}}{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2 \cosh \dfrac{2\bar{\sigma}}{\sigma_0}} \tag{A24b}$$

In equations (A24),

$$r = \tanh\left[\frac{1}{2}\left(\frac{E_1}{E_1 + E_2} \frac{2\bar{\sigma}}{\sigma_0} - \tanh^{-1} \frac{M_2 \sinh \dfrac{2\bar{\sigma}}{\sigma_0}}{M_1 + M_2 \cosh \dfrac{2\bar{\sigma}}{\sigma_0}}\right)\right]$$

$$\tau = \frac{\dfrac{\sigma_0}{\sqrt{M_1 M_2}}\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)}{\sqrt{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2 \cosh \dfrac{2\bar{\sigma}}{\sigma_0}}}$$

Form for Comparison With Data

The creep strain ϵ is usually not plotted against $\dfrac{t}{\tau}$ as suggested by equations (A24) but instead against the parameter $\theta = te^{-\Delta H/RT}$, which in the notation of this paper is

$$\theta = \frac{t}{\tau}\,te^{-\Delta H/RT} = \frac{\dfrac{t}{\tau}\,\dfrac{\sigma_0}{\sqrt{M_1 M_2}}\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)e^{-\Delta H/RT}}{\sqrt{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}}}$$

From equation (A21), $\sqrt{M_1 M_2}$ may be replaced by its value in terms of the steady creep constants. The final result for the parameter θ is

$$\theta = te^{-\Delta H/RT} = \frac{\dfrac{t}{\tau}\,\sigma_0\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)}{2sT\sqrt{\dfrac{M_1}{M_2} + \dfrac{M_2}{M_1} + 2\cosh\dfrac{2\bar{\sigma}}{\sigma_0}}} \qquad (A25)$$

Universal nondimensional transient creep curves can evidently be constructed by using as coordinates the parameters:

For the case $\dfrac{E_1}{E_2} = \infty$,

$$\frac{\epsilon}{\dfrac{\sigma_0}{E_2}}, \qquad \frac{2sT\theta}{\dfrac{\sigma_0}{E_2}}$$

For the case $\dfrac{E_1}{E_2} = 1$,

$$\frac{\epsilon}{\dfrac{\sigma_0}{E}}, \qquad \frac{2sT\theta}{\dfrac{\sigma_0}{E}}$$

or

$$\frac{\epsilon}{\dfrac{\sigma_0}{2E}}, \qquad \frac{2sT\theta}{\dfrac{\sigma_0}{2E}}$$

which provide better separation of the curves in the region of zero transient creep.

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TABLE I

NUMERICAL VALUES OF EXPERIMENTAL AND ASSUMED CONSTANTS

Metal	Steady creep constants from experimental data				Constants assumed for theoretical curves				
	$2sT$, hr^{-1}	σ_0, psi	ΔH, $\dfrac{calorie}{mol}$	Reference	$2sT$, hr^{-1}	σ_0, psi	$\dfrac{\sigma_0}{E_2}$	$\dfrac{M_1}{M_2}$	$\dfrac{E_1}{E_2}$
Aluminum (99.987%)	0.5×10^{11}	382	35,800	5	0.5×10^{11}	382	0.02	10	∞
Gamma iron (0.44% C)	2.5	835	78,000	6,7	2.5	835	.006	100	∞
Lead (coarse-grained)	7.8	96	*25,400	8	7.8	96	.0048	1	∞
7075-T6 aluminum-alloy sheet	1.5	4,300	34,700	1	1.5	4,300	.0004	1,000	1

*This value of ΔH (the mean of the two values determined by Wiseman, Sherby, and Dorn in ref. 9 for single and polycrystals) was used, together with a temperature of 290° K, for computation of the parameter θ from the data of reference 8.

TABLE II

COMPATIBILITY OF IMPLIED VALUES OF $\frac{E_1}{E_2}$ WITH ASSUMED VALUES

Metal	Implied value of $E_2 = \dfrac{\sigma_0}{\dfrac{\sigma_0}{E_2}}$, psi	Approximate modulus, $E = \dfrac{E_1 + E_2}{2}$, psi	Implied value of $\dfrac{E}{E_2}$	Implied value of $\dfrac{E_1}{E_2}$	Assumed value of $\dfrac{E_1}{E_2}$
Aluminum (99.987%)	1.9×10^4	10^7	525	1,049	∞
Gamma iron (0.44% C)	1.4×10^5	3×10^7	214	417	∞
Lead (coarse-grained)	2.0×10^4	5×10^6	250	499	∞
7075-T6 aluminum-alloy sheet	1.1×10^7	10^7	.9	.8	1

(a) $\dfrac{E_1}{E_2} = \infty$.
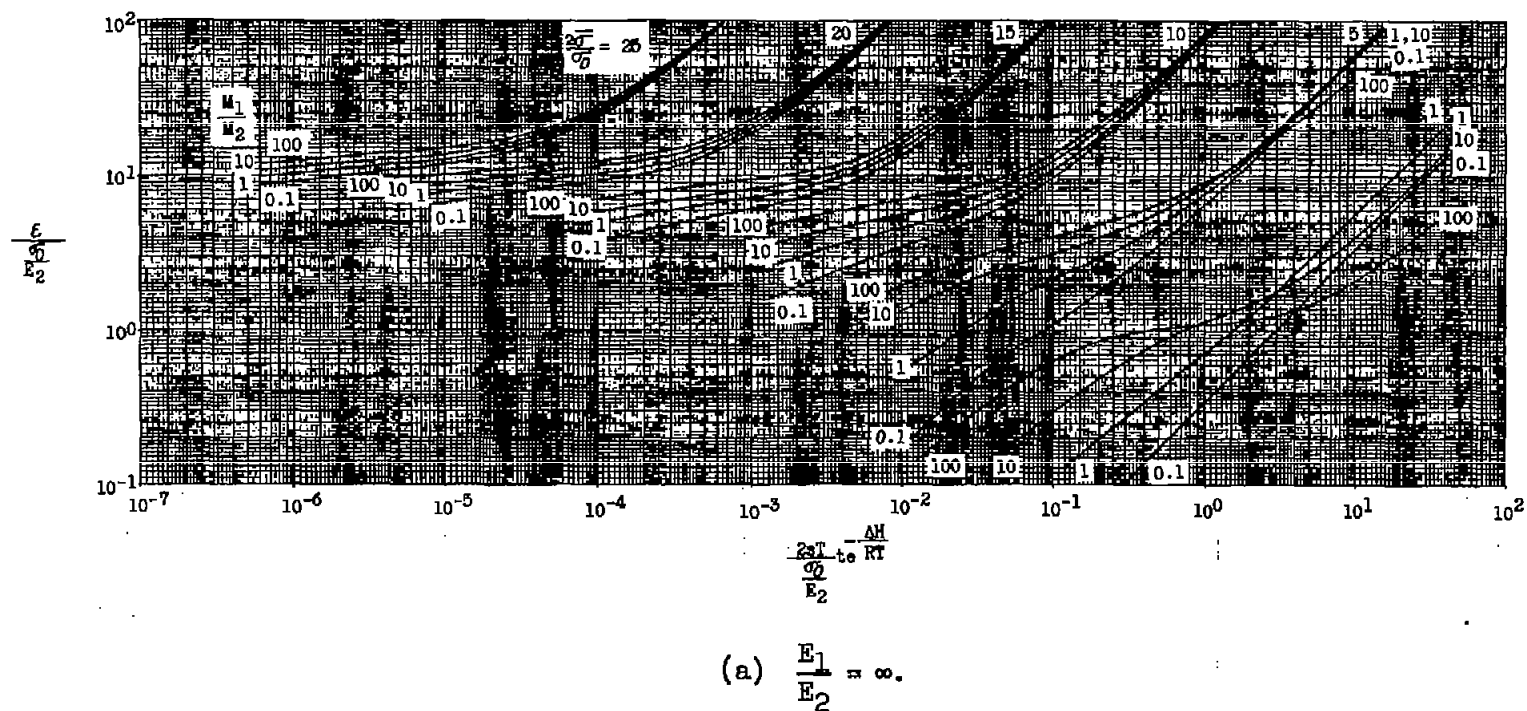
Figure 1.- Universal nondimensional curves for transient creep.
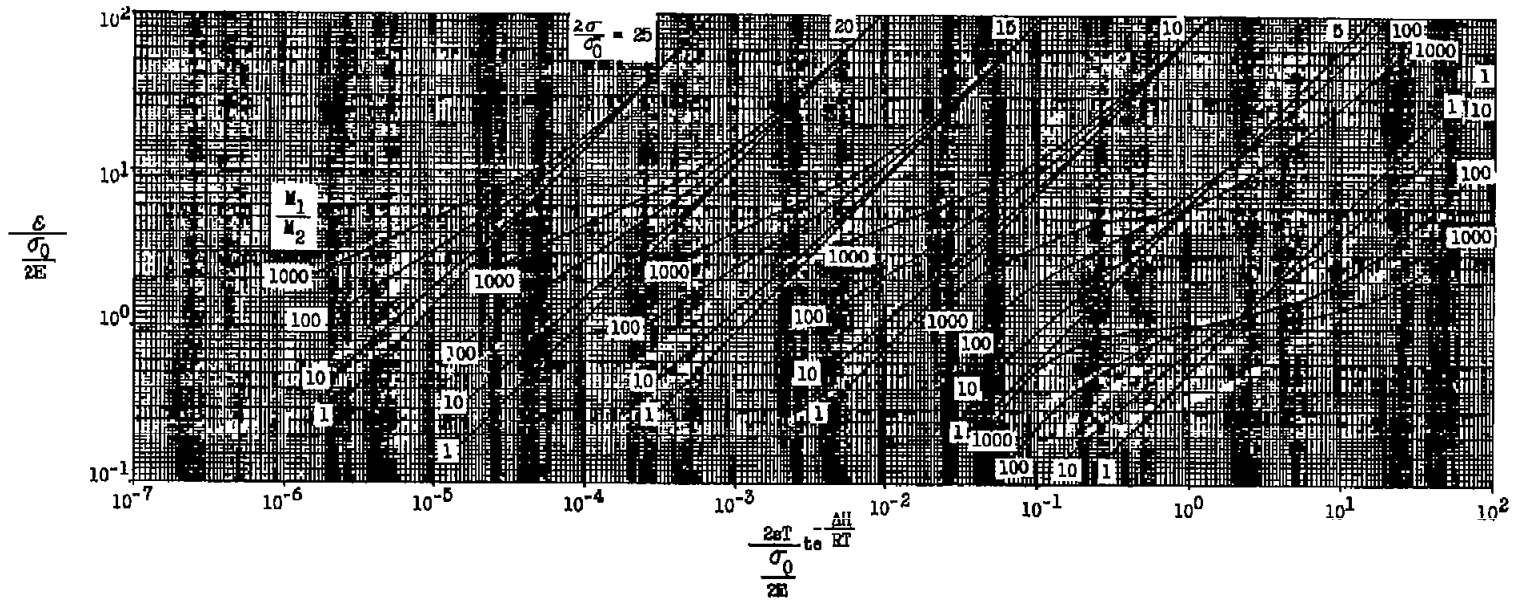
(b) $\dfrac{E_1}{E_2} = 1.$

Figure 1.- Concluded.
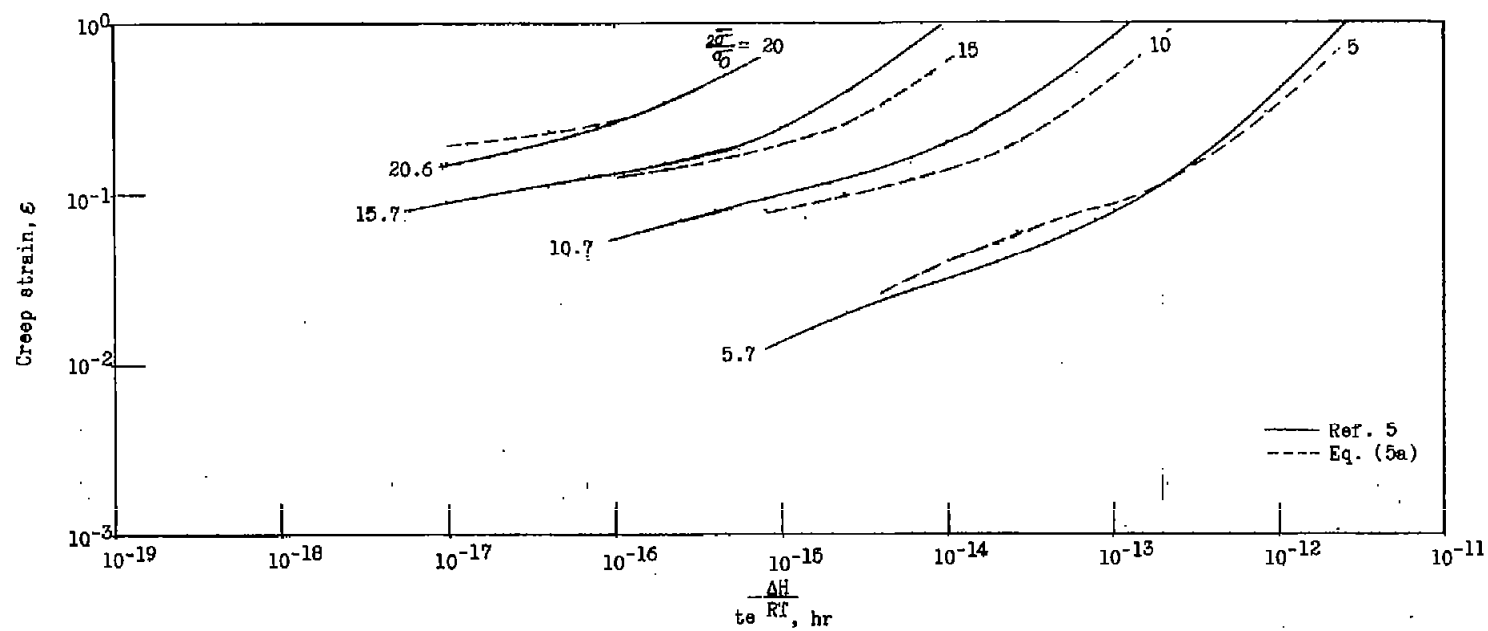
Figure 2.- Comparison of theory and experiment for the transient creep of pure aluminum.
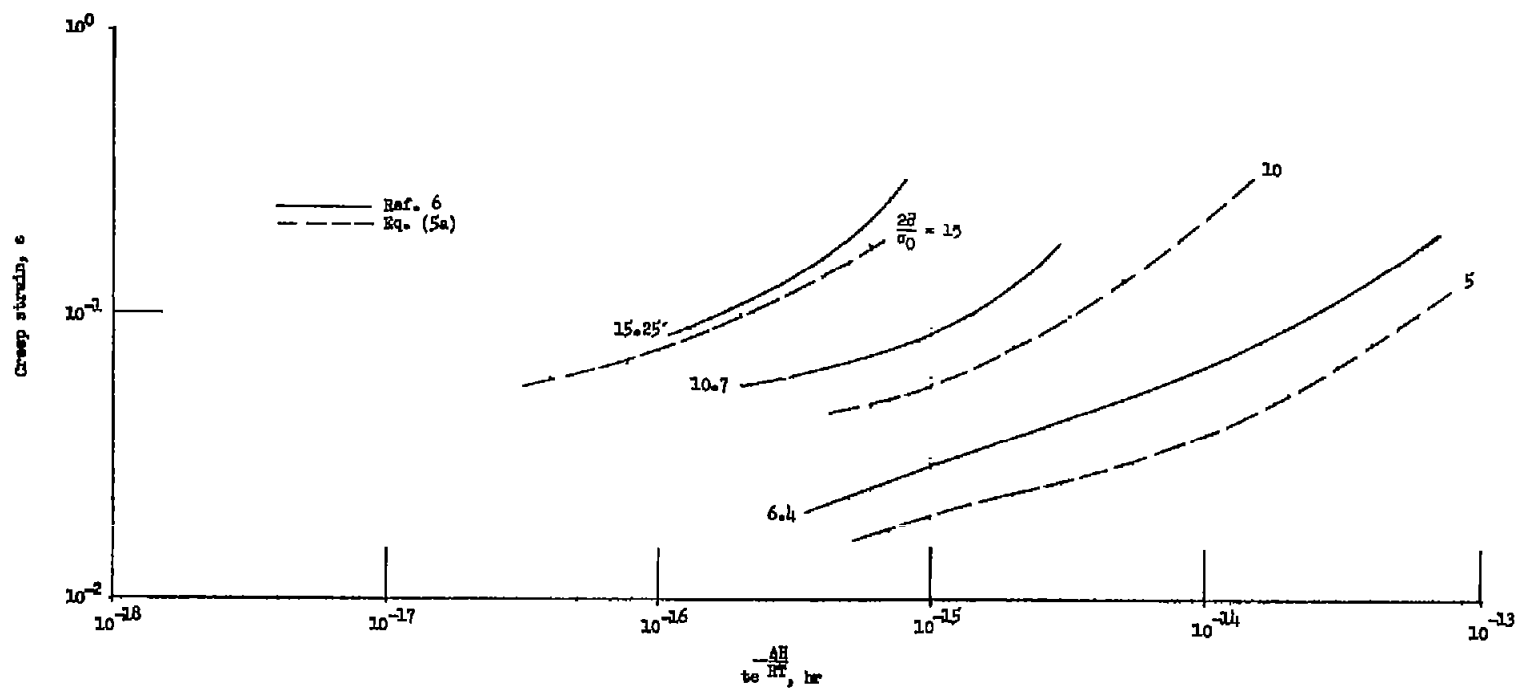
Figure 3.- Comparison of theory and experiment for the transient creep of gamma iron.
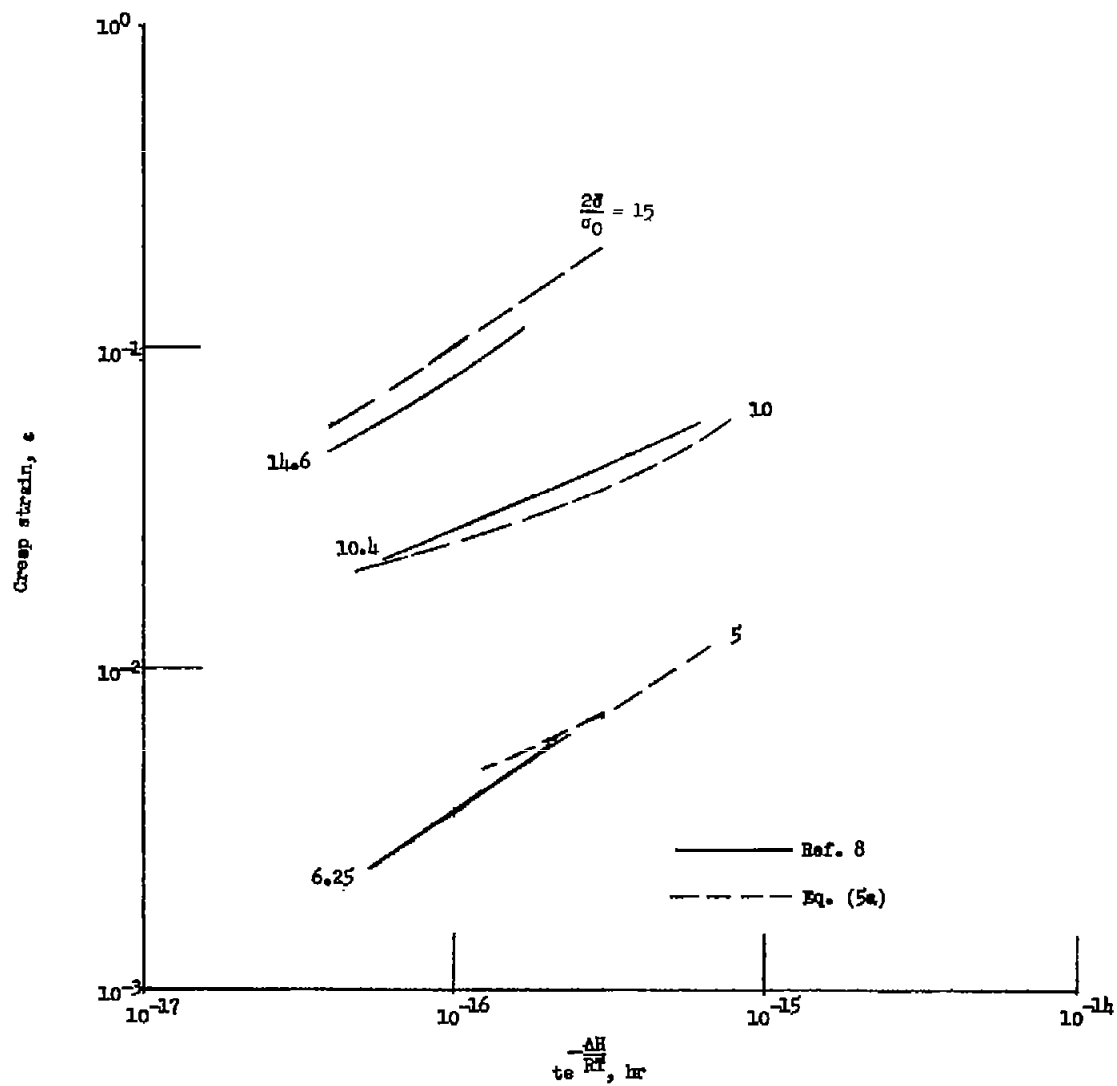
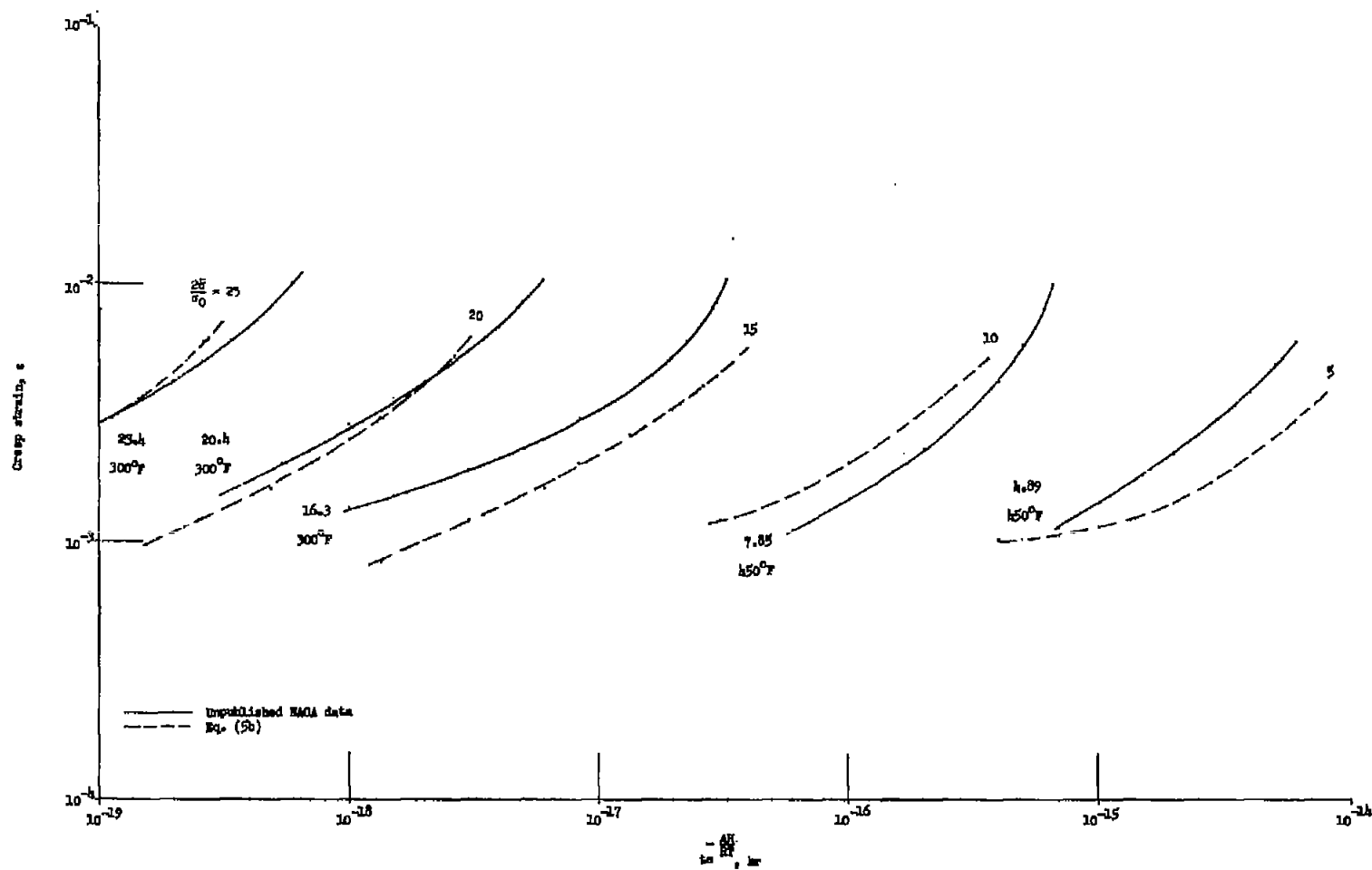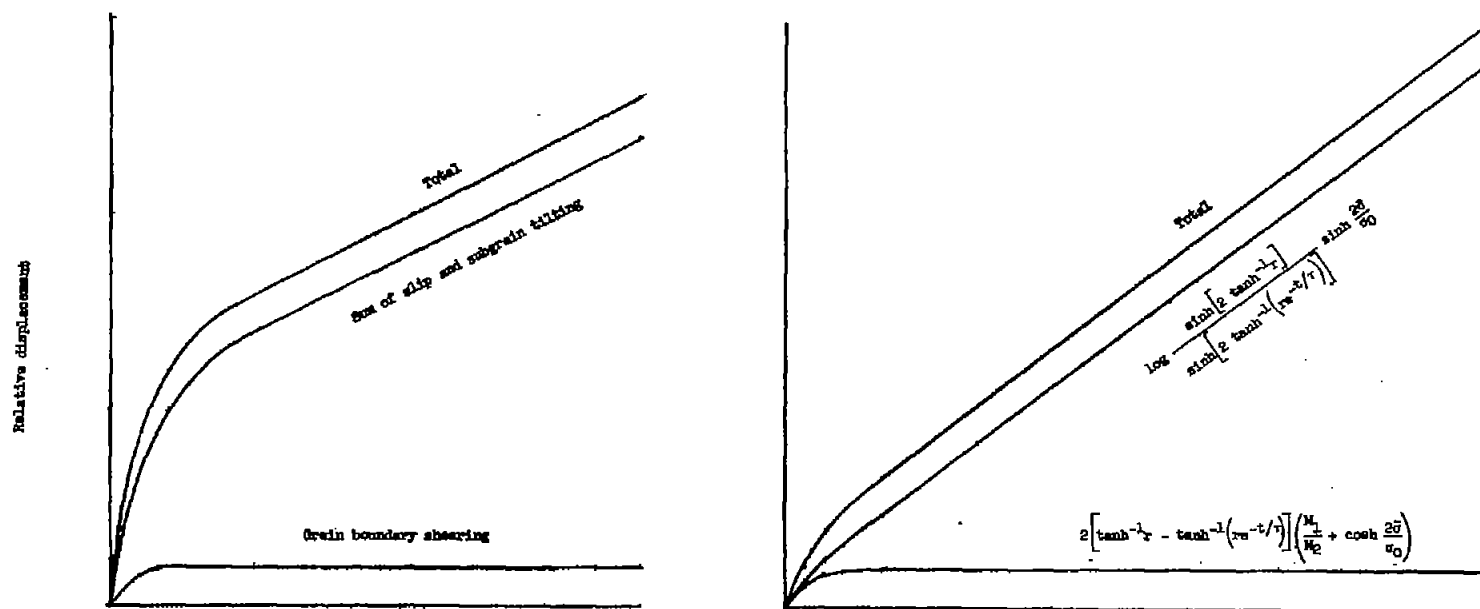Figure 4.- Comparison of theory and experiment for the transient creep of lead.

Figure 5.- Comparison of theory and experiment for the transient creep of 7075-T6 aluminum alloy.

(a) Experimental observations of the indi-
vidual components by McLean (ref. 10).

(b) Contribution of the individual terms
from equation (5a) of this paper.

$$\frac{2\bar{\sigma}}{\sigma_0} = 5; \quad \frac{M_1}{M_2} = 1; \quad \frac{E_1}{E_2} = \infty.$$

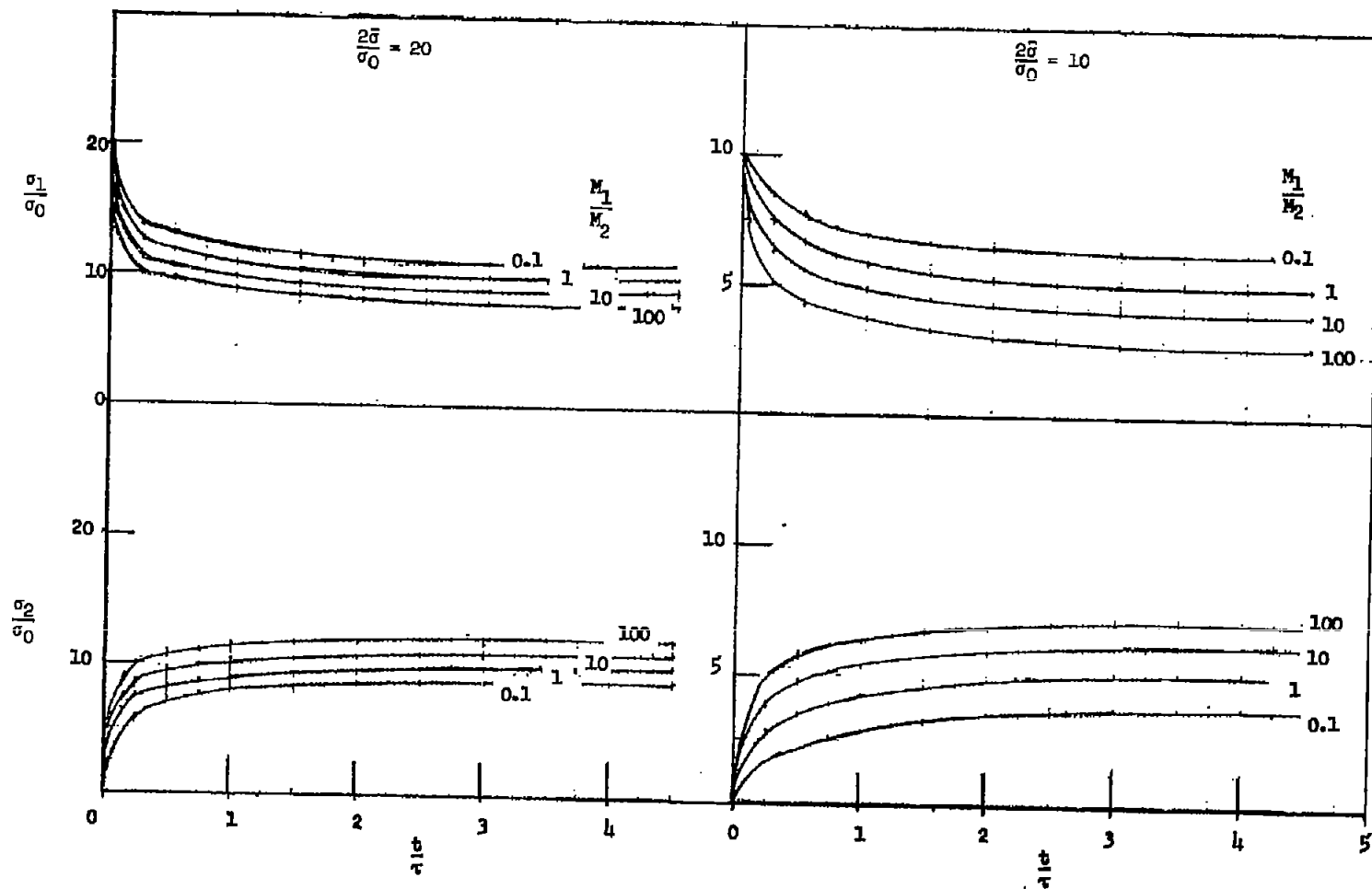Figure 6.- Contribution of individual components to the total transient creep.

Figure 7.- Theoretical distribution of stress between the two phases for two applied stress ratios.